DOCUMENT RESUME

ED 392 817 TM 024 455

AUTHOR	Mislevy, Robert J.; And Others
TITLE	Toward a Test Theory for Assessing Student Understanding.
INSTITUTION	Educational Testing Service, Princeton, N.J.
SPONS AGENCY	Office of Naval Research, Arlington, VA. Cognitive and Neural Sciences Div.
REPORT NO	ETS-RR-91-32-ONR
PUB DATE	Apr 91
CONTRACT	N00014-88-K-0304
NOTE	46p.
PUB TYPE	Viewpoints (Opinion/Position Papers, Essays, etc.) (120) -- Reports - Evaluative/Feasibility (142)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Cognitive Psychology; *Comprehension; *Constructivism (Learning); *Educational Assessment; Educational Practices; Educational Psychology; Item Response Theory; Knowledge Level; Learning Strategies; *Student Evaluation; Student Interests; *Test Theory
IDENTIFIERS	*Chunking

ABSTRACT

The view of learning that underlies standard test theory is inconsistent with the view rapidly emerging from cognitive and educational psychology. Learners become more competent not simply by learning more facts and skills, but by reconfiguring their knowledge; by "chunking" information to reduce memory loads; and by developing strategies and models that help them discern when and how facts and skills are important. Neither classical test theory nor item response theory is designed to inform educational decisions conceived from this perspective. This paper sketches the outlines of a test theory built around models of student understanding, as inspired by the substance and the psychology of the domain of interest. The ideas are illustrated with a simple numerical example based on the balance beam tasks of R. S. Siegler. Directions in which the approach must be developed to be broadly useful in educational practice are discussed. An appendix gives equations for the latent class model. (Contains 2 tables, 8 figures, and 48 references.) (Author/SLD)

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RR-91-32-ONR

TOWARD A TEST THEORY FOR ASSESSING STUDENT UNDERSTANDING

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and
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This research was sponsored in part by the
Program Research Planning Council
Educational Testing Service; and the
Cognitive Science Program
Cognitive and Neural Sciences Division
Office of Naval Research, under
Contract No. N00014-88-K-0304
R&T 4421552

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Educational Testing Service
Princeton, New Jersey

April 1991

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TM024455

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a REPORT SECURITY CLASSIFICATION: Unclassified	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S): RR-91-32-ONR	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: Educational Testing Service; 6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION: Cognitive Science Program, Office of Naval Research (Code 1142CS), 800 North Quincy Street
6c ADDRESS (City, State, and ZIP Code): Princeton, NJ 08541	7b ADDRESS (City, State, and ZIP Code): Arlington, VA 22217-5000
8a NAME OF FUNDING/SPONSORING ORGANIZATION; 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: N00014-88-K-0304
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO: 61153N; PROJECT NO: RR04204; TASK NO: RR04204-01; WORK UNIT ACCESSION NO: R&T4421552

11 TITLE (Include Security Classification)

Toward a Test Theory for Assessing Student Understanding (Unclassified)

12 PERSONAL AUTHOR(S)

Robert J. Mislevy, Kentaro Yamamoto, and Steven Anacker

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Technical	FROM _____ TO _____	April 1991	37

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	
05	10		Inference networks, cognitive assessment, student models, Bayesian inference

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

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20 DISTRIBUTION/AVAILABILITY OF ABSTRACT	21 ABSTRACT SECURITY CLASSIFICATION
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL: Dr. Charles E. Davis	22b TELEPHONE (Include Area Code): 703-696-4046; 22c OFFICE SYMBOL: ONR 1142CS

DD Form 1473, JUN 86 — Previous editions are obsolete — S/N 0102-LF-014-6603

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This work was supported by Contract No. N00014-88-K-0304, R&T 4421552, from the Cognitive Science Program, Cognitive and Neural Sciences Division, Office of Naval Research, and by the Program Research Planning Council of Educational Testing Service. We are grateful to Dr. Steen Andreassen for permission to reproduce Figures 1 and 2, to Robert Siegler for the use of data from his balance-beam studies, and to Larry Frase and Kikumi Tatsuoka for comments on an earlier version of the paper.

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Abstract

The view of learning that underlies standard test theory is inconsistent with the view rapidly emerging from cognitive and educational psychology. Learners become more competent not simply by learning more facts and skills, but by reconfiguring their knowledge; by "chunking" information to reduce memory loads; and by developing strategies and models that help them discern when and how facts and skills are important. Neither classical test theory nor item response theory (IRT) is designed to inform educational decisions conceived from this perspective. This paper sketches the outlines of a test theory built around models of student understanding, as inspired by the substance and the psychology of the domain of interest. The ideas are illustrated with a simple numerical example based on Siegler's balance beam tasks. Directions in which the approach must be developed to be broadly useful in educational practice are discussed.

Background

When schooling became mandatory at the turn of the century, educators suddenly faced selection and placement decisions for unprecedented numbers of students, displaying the diversity of abilities and backgrounds that individuals bring to schooling (Glaser, 1981). Numbers of correct answers to multiple-choice test items were used to rank students according to their overall proficiencies in domains of tasks. These rankings were used in turn to predict students' success in fixed educational experiences.

Classical test theory (CTT) emerged when Spearman (e.g., 1907) applied statistical methods to study how reliable estimates of this overall proficiency would be from different test forms that might be constructed for the purpose. Extensions of this work led over the years to a vast armamentarium of techniques for building tests and making decisions with test scores (Gulliksen, 1950); to an axiomatic foundation for statistical inference about test scores (Lord, 1959; Lord & Novick, 1968; Novick, 1966); and to sophisticated techniques for partitioning test score variance according to facets of items, persons, and observational settings (Cronbach, Gleser, Nanda, & Rajaratnam,1972). It is important to note that in all this work, the object of inference is overall proficiency—the test score, observed or expected— in terms of numbers of correct responses in a domain of items.

Item response theory (IRT; see Hambleton, 1989, for an overview) represented a major practical advance over CTT by modeling probabilities of correct item response in terms of an unobservable proficiency variable. IRT solves many problems that were difficult under CTT, in equating, test construction, and adaptive testing. Advanced statistical methods have been brought to bear on inferential problems in IRT, including sophisticated estimation algorithms (e.g., Bock & Aitkin, 1981), techniques from missing-data theory (Mislevy, in press-a), and Bayesian treatments of uncertainty in models and parameters (Lewis, 1985; Mislevy & Sheehan, 1990; Tsutakawa & Johnson, 1988). The underlying psychological model remains quite simple, however; as in CTT, the focus remains on overall proficiency in a domain of items. From the perspective of IRT, two students with the same overall proficiency are indistinguishable.

As useful as standard tests and standard test theory have proven in large-scale evaluation, selection, and placement problems, their focus on *who* is competent and *how many* items they answer can fall short when the goal is to improve individuals' competencies. Glaser, Lesgold, and Lajoie (1987) point out that tests can predict failure

without an understanding of what causes success, but intervening to prevent failure and enhance competence requires deeper understanding.

The past decade has witnessed considerable progress toward the requisite understanding. Psychological research has moved away from the traditional laboratory studies of simple (even random!) tasks, to tasks that better approximate the meaningful learning and problem-solving activities that engage people in real life. Studies comparing the ways experts differ from novices in applied problem-solving in domains such as physics and trouble-shooting (e.g., Chi, Feltovich & Glaser, 1981) reveal the central importance of knowledge structures—networks of concepts and interconnections among them—that impart meaning to patterns in what one observes and how one chooses to act. The process of learning is to a large degree expanding these structures and, importantly, *reconfiguring them* to incorporate new and qualitatively different connections as the level of understanding deepens. Educational psychologists have begun to put these findings to work in designing both instruction and tests (e.g., Glaser et al., 1987; Greeno, 1976; Marshall, 1985, in press). Again in the words of Glaser, Lesgold, and Lajoie (1987),

> "Achievement testing as we have defined it is a method of indexing stages of competence through indicators of the level of development of knowledge, skill, and cognitive process. These indicators display stages of performance that have been attained and on which further learning can proceed. They also show forms of error and misconceptions in knowledge that result in inefficient and incomplete knowledge and skill, and that need instructional attention." (p.81)

Paraphrasing Ohlsson and Langley (1985), Clancey (1986) summarizes the shift in perspective: "[to] describing mental processes, rather than quantifying performance with respect to stimulus variables; describing individuals in detail, not just stating generalities; and giving psychological interpretation to qualitative data, rather than statistical treatment to numerical measurements" (p. 391).

An Approach to Modeling Student Understanding

The modeling approach we are beginning to pursue can be encapsulated as follows:

> "Standard test theory evolved as the application of statistical theory with a simple model of ability that suited the decision-making environment of mass educational systems. Broader educational options, based on insights into

> the nature of learning and supported by more powerful technologies, demand a broader range of models of capabilities—still simple compared to the realities of cognition, but capturing patterns that inform a broader range of instructional alternatives. A new test theory can be brought about by applying to well-chosen cognitive models the same general principles of statistical inference that led to standard test theory when applied to the simple model." (Mislevy, in press-b).

The approach begins in a specific application by defining a universe of student models. This "supermodel" is indexed by parameters that signify distinctions between states of understanding. Symbolically, we shall refer to the (typically vector-valued) parameter of the student-model as η. A particular set of values of η specifies a particular student model, or one particular state among the universe of possible states the supermodel can accommodate. These parameters can be qualitative or quantitative, and qualitative parameters can be unordered, partially ordered, or completely ordered. A supermodel can contain any mixture of these types. Their nature is derived from the structure and the psychology of the learning area, the idea being to capture the essential distinctions among students.

Any application faces a modeling problem, an item construction problem, and an inference problem.

The *modeling* problem is delineating the states or levels of understanding in a learning domain. In meaningful applications this might address several distinct strands of learning, as understanding develops in a number of key concepts, and it might address the connectivity among those concepts.[1] Symbolically, this substep defines the *structure* of $p(\mathbf{x}|\eta)$, where $\mathbf{x}$ represents observations. Obviously any model will be a gross simplification of the reality of cognition. A first consideration in what to include in the supermodel is the substance and the psychology of the domain: Just what are the key

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[1] A particularly interesting special case occurs when the universe of student models can be expressed a performance models (Clancey, 1986). A performance model consists a knowledge base and manipulation rules that can be run on problems in a domain of interest. A particular model can contain both knowledge and production rules that are incorrect or incomplete; the solutions it produces will be correct or incorrect in identifiable ways. Here the parameter η specifies features of performance models.

concepts? What are important ways of understanding and misunderstanding them? What are typical paths to competence? A second consideration is the so-called grain-size problem, or the level of detail at which student-models should differ. A major factor in answering this question is the decision-making framework under which the modeling will take place. As Greeno (1976) points out, "It may not be critical to distinguish between models differing in processing details if the details lack important implications for quality of student performance in instructional situations, or the ability of students to progress to further stages of knowledge and understanding."

The *item construction* problem is devising situations for which students who differ in the parameter space are likely to behave in observably different ways. The conditional probabilities of behavior of different types given the unobservable state of the student are the *values* of $p(\mathbf{x}|\eta)$, which may in turn be modeled in terms of another set of parameters, say β. The $p(\mathbf{x}|\eta)$ values provide the basis for inferring back about the student state. An element in $\mathbf{x}$ could contain a right or wrong answer to a multiple-choice test item, but it could instead be the problem-solving approach regardless of whether the answer is right or wrong, the quickness of a responding, a characteristic of a think-aloud protocol, or an expert's evaluation of a particular aspect of the performance. The effectiveness of an item is reflected in differences in conditional probabilities associated with different parameter configurations, so an item may be very useful in distinguishing among some aspects of potential student models but useless for distinguishing among others. Tatsuoka (1989) demonstrates the relationship between item construction and inference about students' strategies for subtracting mixed numbers.

The *inference* problem is reasoning from observations to student models. The model-building and item construction steps provide η and $p(\mathbf{x}|\eta)$. Let $p(\eta)$ represent expectations about η in a population of interest—possibly non-informative, possibly based on expert opinion or previous analyses. Bayes theorem can be employed to draw inferences about η given $\mathbf{x}$ via $p(\eta|\mathbf{x}) \propto p(\mathbf{x}|\eta)\, p(\eta)$. Thus $p(\eta|\mathbf{x})$ characterizes belief about a particular student's model after having observed a sample of the student's behavior. Practical problems include characterizing what is known about β so as to determine $p(\mathbf{x}|\eta)$, carrying out the computations involved in determining $p(\eta|\mathbf{x})$, and, in some applications, developing strategies for efficient sequential gathering of observations. As we have noted, analogous problems have been studied in standard test theory, and the solutions there, because they are applications of general principles of statistical inference, generalize to

models built around alternative psychological models. The models are more realistic and more ambitious, but the formalism is identical.[2]

Previous Research

Research relevant to this approach has been carried out in a wide variety of fields, including cognitive psychology, the psychology of mathematics and science education, artificial intelligence (AI) work on student modeling, test theory, and statistical inference. Cognitive scientists have suggested general structures such as "frames" or "schemas" that can serve as a basis for modeling understanding (e.g., Minsky, 1975; Rumelhart, 1980), and have begun to devise tasks that probe their features (e.g., Marshall, 1989, in press). Researchers interested in the psychology of learning in subject areas such as proportional reasoning have focused on identifying key concepts, studying how they are typically acquired (e.g., in mechanics, Clement, 1982; in ratio and proportional reasoning, Karplus, Pulos, & Stage, 1983), and constructing observational settings that allow one to infer students' understanding (e.g., van den Heuvel, 1990; McDermott, 1984). We make no effort here to review these literatures, but point out that our work can succeed only by building upon their foundations. Our potential contribution would be to the structures and mechanics of model-building and inference. The following sections briefly mention some important work along these lines from test theory and statistics.

Modeling Student Behavior

The standard models of educational measurement are concerned solely with examinees' tendencies to answer items correctly—that is, their overall proficiency. Recently, however, models that focus on patterns other than overall proficiency have begun to appear the test theory literature. Some examples that are relevant to educational applications are listed below.

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[2] Advocates of student modeling emphasize the qualitative aspects of student models. Our approach is compatible with this view, as it is possible to build universes of qualitative models, indexed by parameters that distinguish their features. Our knowledge about a particular student's model is imperfect, however. It can be expressed in terms of probabilities expressing the plausibility of various models, given what has been observed. Probabilities are quantitative, and admit to a calculus of manipulation. We might thus employ a *quantitative* model for our (imperfect) knowledge about *qualitative* student models.

1. Mislevy and Verhelst's (1990) *mixture models* for item responses when different examinees follow different solution strategies or use alternative mental models. When a single IRT model cannot capture key distinctions among examinees, it may suffice to posit qualitatively distinct classes of examinees and use IRT models to summarize distinctions among examinees within these classes.

2. Wilson's (1989b) *Saltus* model for characterizing stages of conceptual development. This model parameterizes the differential patterns of strength and weakness expected as learners progress through successive conceptualizations of a domain.

3. Falmagne's (1989) and Haertel's (1984) latent class models for *Binary Skills*. These models are intended for domains in which competence can be described by the presence or absence of several (possibly complex) elements of skill or knowledge, and observational situations can be devised that demand various combinations of these skills. Also see Paulson (1986) for an alternative use of latent class modelling in cognitive assessment.

4. Embretson's (1985) *multicomponent models* for integrating item construction and inference within a unified cognitive model. The conditional probabilities of solution steps given a multifaceted student model are given by IRT-like statistical structures.

5. Tatsuoka's (1989) *Rule space* analysis. Tatsuoka uses a generalization of IRT methodology to define a metric for classifying examinees based on likely patterns of item response given patterns of knowledge and strategies.

6. Yamamoto's (1987) *Hybrid* model for dichotomous responses. The *Hybrid* model characterizes an examinee as either belonging to one of a number of classes associated with states of understanding, or in a catch-all IRT class. This approach might be useful when certain response patterns signal states of understanding for which particular educational experiences are known to be effective. Instructional decisions are triggered by these patterns if they are detected, but by overall proficiency when no more targeted action can be provided.

7. Masters and Mislevy's (in press) and Wilson's (1989a) use of the *Partial Credit* rating scale model to characterize levels of understanding, as evidenced by the nature or approach of a performance rather than its correctness. These applications incorporate into a

probabilistic framework the cognitive perspective underlying Biggs and Collis's (1982) SOLO taxonomy for describing salient qualities of performances.

These are the rudiments of models upon which concept-referenced achievement measures can be based. Applications to date have been fairly limited, and most have addressed one-to-many relationships between an underlying knowledge state and observable behavior. That is, a single (possibly unordered or multifaceted) variable has been used to characterize examinees, and performance on all items is modeled in terms of this variable. What is lacking from the point of view of the educator is the fact that meaningful real world tasks are rarely segregated into these neat little sets. Rather, they often involve multiple concepts, connections among larger concepts, and transformations among alternative representations of a domain. While the simple tasks that characterize one-to-many domains are essential at early stages of learning, more complex tasks that involve multiple concepts in many-to-many relationships are needed to promote the integration among concepts that form the core of what is often called "higher-level learning."

Inference Networks

Recent developments in the context of probability-based inference networks (Lauritzen & Spiegelhalter, 1988; Pearl, 1988) offer a capability for integrating conceptual models of the type described above. These probability-based structures are attractive for educational measurement because they permit a coherent extension of the modeling approach and inferential logic of the new cognitive-assessment models mentioned above. To show how the approach might be applied in the educational setting, we first discuss an application in the setting of medical diagnosis.

MUNIN is an inference network that organizes knowledge in the domain of electromyography—the relationships among nerves and muscles. Its function is to diagnose nerve/muscle disease states. The interested reader is referred to Andreassen, Woldbye, Falck, and Andersen (1987) for a fuller description. The prototype discussed in that presentation and used for our illustration concerns a single arm muscle, with concepts

represented by twenty-five nodes and their interactions represented by causal links.[3] A graphic representation of the network appears in Figure 1.

[Figure 1 about here]

The rightmost column of nodes in Figure 1 concerns outcomes of potentially observable variables, such as symptoms or test results. These outcomes are the **x** vector in our earlier notation. The middle layers are "pathophysiological states," or syndromes. These drive the probabilities of observations. The leftmost layer is the underlying disease state, including three possible diseases in various stages, no disease, or "Other"—a condition not built into in the system. These states drive the probabilities of syndromes. It is assumed that a patient's true state can be adequately characterized by values of these disease and syndrome states—our η parameter. Paths indicate conditional probability relationships, which are to be determined either logically, subjectively, purely empirically, or through model-based statistical estimation. In particular, the paths ending at observables represent $p(\mathbf{x}|\eta)$. Note that the probabilities of observables depend on some syndromes, but not others. The lack of a path signifies conditional independence. Note also that a given test result can be caused by different disease combinations.

As a patient enters the clinic, the diagnostician's state of knowledge about him is expressed by population base rates, or $p(\eta)$. This is depicted in Figure 1 by bars that represent the base probabilities of disease and syndrome states. Base rates of observable test results are similarly shown. Tests are carried out, one at a time or in clusters, and with each result the probabilities of disease states are updated. The expectations of tests not yet given are calculated, and it can be determined which test will be most informative in identifying the disease state. Knowledge is thus accumulated in stages, from $p(\eta)$ to $p(\eta|\mathbf{x}_1)$ after observing the first subset of tests, to $p(\eta|\mathbf{x}_1,\mathbf{x}_2)$ after the second, and so on, with each successive test selected optimally in light of knowledge at that point in time. Figure 2 illustrates the state of knowledge after a number of electromyographic test results have been observed. Observable nodes with results now known are depicted with shaded bars representing observed values For them, knowledge is perfect. The implications of these results have been propagated leftward to syndromes and disease states, as shown by

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[3] The ESPRIT team has generalized the application to address clusters of interrelated muscles in a network containing over a thousand nodes.

distributions that differ from the base rates in Figure 1. These values guide the decision to test further or initiate a treatment. Finally, updated beliefs about disease states have been propagated back toward the right to update expectations about the likely outcomes of test not yet administered. These expectations, and the potential they hold for further updating knowledge about the disease states, guide the selection of further tests.

[Figure 2 about here]

Inference Networks in the Educational Setting

To see how the ideas underlying MUNIN apply to the educational setting, consider the following analogy:

Medical Application	Educational Application
Observable symptoms, medical tests	Test items, verbal protocols, teachers' ratings of levels of understanding, solution traces
Disease states, syndromes	States or levels of understanding of key concepts, available strategies
Architecture of interconnections based on medical theory	Architecture of interconnections based on cognitive and educational theory
Conditional probabilities given by physiological models, empirical data, expert opinion	Conditional probabilities given by psychological models, empirical data, expert opinion

The definitions of key concepts will be guided by theorized and observed stages of learning in the area, and the connections with observables will be expressed through measurement models such as those discussed above. The initialization of the probabilities in the network will be accomplished by one or more methods: clinical analysis, with skilled interviewers assessing in detail the nature of students' understandings and related these understandings to task performances; statistical analysis of data concerning selected models

for portions of the larger network (Mislevy & Verhelst, 1990); or theoretical analysis, in which logic or theory provides expectations for outcomes under hypothesized cognitive states. After the initialization phase, connections can be updated periodically with the larger amounts of less precise data that will be accumulated as students provide information about the adequacy of the relationships embodied in the network and the accuracy of the baseline and conditional probabilities.

A Numerical Example

Siegler's balance beam tasks

Kuhn (1970) emphasizes the central role that exemplars, or small, archetypical examples, play in science. Textbook examples are the vehicle through which students are acculturated to the concepts and relationships of a particular way of viewing a class of phenomena—a paradigm, in Kuhn's words. They function almost like parables or morality tales. New paradigms are introduced with new exemplars, that introduce new concepts, highlight differences between the new paradigm and the old, and demonstrate how the new way of thinking solves problems the old way could not. Modeling the states of the electron in the hydrogen atom possesses this status in quantum mechanics. Explaining children's understanding of balance beam problems, an exemplar from developmental psychology originated by Piaget, is approaching the same status in test theory (e.g., Kempf, 1983, Mislevy, in press-b, and Wilson, 1989b). Robert Siegler's balance beam tasks yield data that are, on the surface, indistinguishable from standard test data, but there are two key distinctions:

1. What is important about examinees is not their overall probability of answering items correctly, but their (unobservable) state of understanding of the domain.

2. Children at less sophisticated levels of understanding initially get certain problems right for the wrong reasons. These items are more likely to be answered wrong at intermediate stages, as understanding deepens! They are bad items by the standards of classical test theory and IRT, because probabilities of correct response do not increase monotonically with increasing total test score. From the perspective of the developmental theory, however, not only is this reversal expected, but it plays an important role in distinguishing among children with different ways of thinking about the problems.

Attempting to study children's reasoning in a manner less subjective than Piaget's unstructured interviews, Siegler (1981) devised a series of balance beam tasks like the one illustrated in Figure 3. Varying numbers of weights are placed at varying locations on a balance beam. The child predicts whether the beam will tip to left, to the right, or remain in balance. Piaget's analysis of children's behavior on balancing tasks (Inhelder & Piaget, 1958), posits that a child will respond in accordance with his or her stage of understanding. The usual stages through which children progress can be described in terms of successive acquisition of the rules listed below.

[Figure 3 about here]

Rule I: If the weights on both sides are equal, it will balance. If they are not equal, the side with the heavier weight will go down. (Weight is the "dominant dimension," because children are generally aware that weight is important in the problem earlier than they realize that distance from the fulcrum, the "subordinate dimension," also matters.)

Rule II: If the weights and distances on both sides are equal, then the beam will balance. If the weights are equal but the distances are not, the side with the longer distance will go down. Otherwise, the side with the heavier weight will go down. (A child using this rule uses the subordinate dimension only when information from the dominant dimension is equivocal.)

Rule III: Same as Rule II, except that if the values of both weight and length are unequal on both sides, the child will "muddle through" (Siegler, 1981, p.6). (A child using this rule now knows that both dimensions matter, but doesn't know just how they combine. Responses will be based on a strategy such as guessing.)

Rule IV: Combine weights and lengths correctly (i.e., compare torques, or products of weights and distances).

It was thus hypothesized that each child could be classified into one of five stages—the four characterized by the rules, or an earlier "preoperational" stage in which neither weight nor length are thought to bear any systematic relationship to the action of the beam.

Siegler developed six types of problems listed below to distinguish among children at different stages of reasoning. (See Figure 4 for an example of each.)

Equal problems (E), with matching weights and lengths on both sides.

Dominant problems (D), with unequal weights but equal lengths.

Subordinate problems (S), with unequal lengths but equal weights.

Conflict-dominant problems (CD), in which one side has greater weight, the other has greater length, and the side with the heavier weight will go down.

Conflict-subordinate problems (CS), in which one side has greater weight, the other has greater length, and the side with the greater length will go down.

Conflict-equal problems (CE), in which one side has greater weight, the other has greater length, and the beam will balance.

[Figure 4 about here]

Table 1 shows the probabilities of correct response that would be expected from groups of children in different stages, if their responses were in complete accordance the hypothesized rules. Scanning across the rows reveals how the probability of a correct response to a given type of item does not always increase as level of understanding increases. For example, Stage II children tend to answer CD items right for the wrong reason, while Stage III children, now aware of a conflict, flounder.

[Table 1 about here]

A latent class model for balance beam tasks

If the theory were perfect, the columns in Table 1 would give probabilities of correct response to the various types of items from children at different stages of understanding. Observing a correct response to an S item, for example, would eliminate the possibility that the child was in Stage I. But because the model is not perfect[4], and because children make slips and lucky guesses, any response could be observed from a child in any stage. A latent class model (Lazarsfeld, 1950) can be used to express the

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[4] This model assumes that the five states are exhaustive and mutually exclusive. Alternative models, such as those of Tatsuoka and Yamamoto mentioned earlier, could be used to relax these restrictions.

structure posited in Table 1 while allowing for some "noise" in real data (see Appendix for details). Instead of expecting incorrect responses with probability one to S items from Stage I children, we might posits some small fraction of correct answers—p(S correct| Stage=I). Similar probabilities of "false positives" can be estimated for other cells in Table 1 containing 0's. In the same spirit, probabilities less than one, due to "false negatives," can be estimated for the cells with 1's. Note that inferences cannot be as strong when these uncertainties are present; a correct response to an S item still suggests that a child is probably not in Stage I, but no longer is it proof positive.

Expressing this model in the notation introduced above, η represents stage membership, $\mathbf{x}$ represents item responses, and $p(\mathbf{x}|\eta)$ are conditional probabilities of correct responses to items of the various types from children in different stages—a noisy version of Table 1. The proportions of children in a population of interest at the different stages are $p(\eta)$, and the probabilities that convey our knowledge about a child's stage after we have observed his responses are $p(\eta|\mathbf{x})$.

Siegler created a 24-task test comprised of four tasks of each type. He collected data from 60 children, from age 3 up through college age, at two points in time, for a total of 120 response vectors. We fit a latent class model to these data using the HYBRIL computer program (Yamamoto, 1987), obtaining the conditional probabilities—$p(\mathbf{x}|\eta)$—shown in Table 2, and the following vector summarizing the (estimated) population distribution of stage membership:

$$p(\eta) = (\text{Prob(Stage=0)}, \text{Prob(Stage=I)}, ..., \text{Prob(Stage=IV)})$$

$$= (.257,.227,.163,.275,.078) .$$

[Table 2 about here]

Note that different types of items are differentially useful to distinguish among children at different levels. E items, for example, are best for distinguishing Stage 0 children from everyone else. CD items, which would be dropped from standard tests because their probabilities of correct response do not have a strictly increasing relationship with total scores, help differentiate among children at Stages II, III, and IV.

Figure 5 depicts the state of knowledge about a child before observing any responses using the conventions of the MUNIN figures. Just one item of each type is shown rather than all four for simplicity. The corresponding status of an observable node

(i.e., an item type) is the expectation of a correct response from a child selected at random from the population. The path from the stage-membership node to a particular observable node represents a row of Table 2.

[Figure 5 about here]

Adaptive testing

Figure 5 represents the state of our knowledge about a child's reasoning stage and expected responses before any actual responses are observed. How does knowledge change when a response is observed? One of the children in the sample, Douglas, gave an incorrect response to his first S item. This could happen regardless of Douglas' true stage; the probabilities are obtained by subtracting the entries in the S row of Table 2 from 1.000, yielding, for Stages 0 through IV, .667, .973, .116, .019, and .057 respectively. This is the likelihood function for η induced by the observation of the response. The bulk of the evidence is for Stages 0 and I. Combining these values with the initial stage probabilities $p(\eta)$ via Bayes theorem yields updated stage probabilities, $p(\eta|$incorrect response to an S item): for Stages 0 through IV respectively, .41, .52, .04, .01, and .01. Expectations for items not yet administered also change. They are averages of the probabilities of correct response expected from the various stages, now weighted by the new stage membership probabilities. The state of knowledge after observing Douglas' first response is depicted in Figure 6 (see Appendix for details; also see Macready & Dayton, 1989.)

[Figure 6 about here]

In a simulation of adaptive testing, we updated our knowledge about Douglas one response at a time, at each step looking at his actual response to an item expected to most substantially reduce our uncertainty about his stage membership. Figure 7 charts probabilities of stage membership for Douglas after each of the first ten items, showing that we quickly converge to Stage 0.

[Figure 7 about here]

Extending the paradigm

The balance beam exemplar illustrates the challenge of inferring states of understanding, but it addresses development of only a single key concept. A major thrust of our proposal is to characterize interconnections among distinct lines of development.

This section takes a small step in this direction by discussing a hypothetical extension to the exemplar, namely, the ability to carry out the arithmetic operations needed to calculate torques. For illustrative purposes, we simply posit a skill to carry these calculations out reliably, either possessed by a child or not. Obviously states of understanding could be developed in greater detail here.

Calculating and comparing torques to solve the "conflict" problems characterizes Stage IV. But if a child at Stage IV cannot carry out the calculations reliably, his pattern of correct and incorrect responses would be hard to distinguish from that of a child in Stage III. Although the two children might answer about the same number of items correctly, the instruction appropriate for them would differ dramatically. And children at any stage of understanding of the balance beam might be able to carry out the computational operations in isolation. The goal of the extended system is to infer both balance-beam understanding and computational skill. To make the distinctions among states of understanding in this extended domain, we introduce two new types of observations:

1. Items isolating computation, such as "Which is greater, 3x4 or 5x2?"

2. Probes for introspection about solutions to conflict items: "How did you get your answer?"

Figure 8 offers one possible structure for this network. Others could be entertained, and in practice one would compare the degree to which they accord with observed data. To keep the diagram simple, only one balance-beam task each for an S and a CS task are illustrated. E and D items would have the same paths as the S task, and CD and CE tasks would have the same paths as the CS tasks. Also, the paths from Stage 0, I, and II indicators to balance beam tasks are not drawn in. The *structure* of paths, but not necessarily the *values*, would be the same as those connecting the Stage III indicator to those tasks.

[Figure 8 about here]

There are three kinds of unobservable variables in the system. The first group expresses level of understanding in the balance beam domain. It proves convenient to express stage membership in terms of dichotomous indicator variables for each stage, because of the special relationship of Stage IV to computational skill. Second is the ability to carry out the calculations involved in computing torques. The third concerns the integration of balance-beam understanding and calculating proficiency. Specifically, we

posit an indicator for whether a child both is in Stage IV *and* possesses the requisite computational skills. Other features of the network worth mentioning are as follows.

1. The probabilities of the pure computation items depend on the unobservable computation variable only; they are conditionally independent of level of balance beam understanding.

2. The correctness aspect of an answer has only two possibilities, right or wrong, but an explanation can fall into five categories corresponding to levels of understanding. A Stage III child might give an explanation consistent with Stages 0, I, II, or III, but would not give a Stage IV explanation. Theory thus posits that the conditional probability of a Stage K response from a Stage J child is zero if K>J. Conditional probabilities for K≤J might be estimated from data or based on experts' experience. It may turn out, for example, that the most likely explanation for an E task from people at Stage IV would probably be a Stage II explanation: "It balances because both the weights and distances are equal."

3. For children in Stages 0 through III, both the right/wrong answers and the "How" answers to balance beam tasks depend only on level of understanding. Because they do not realize the connection between the problems and the torque calculations, their responses to the balance beam tasks are conditionally independent of their computational skill, even on items for which that skill is an integral component of an expert solution.

4. For children in Stage IV, right/wrong answers to conflict items depend on the understanding/computation integration variable, but "How" answers depend only on understanding. A child in Stage IV with low computational skill can thus be differentiated from a child in Stage III by his higher probabilities of giving Stage IV explanations and incorrect answers to pure computation problems.

Discussion

This conceptual framework described above holds the promise of extending and clarifying standard educational measurement practices in several ways:

Connections with instruction can be forged more easily than with standard tests, because the focus is no longer on *how many* questions a student can answer, but *how* they answer them. In medical diagnosis, different diseases gave rise to similar results in certain

tests; in education, so too can different approaches lead to similar test scores for students. But accounting for the patterns of performance, especially if probing adaptively, can pinpoint the areas which need attention to best improve performance.

Student reports can be provided at varying levels and highlighting different features of a student's status. Of particular importance to the student and the teacher are reports in terms of levels or stages of understanding of key concepts, since this is the level at which instruction is aimed. For the quality control purposes of administrators, however, one could predict a student's performance on a standard set of tasks in the domain—say, a "market basket" of tasks that, ideally, every student should eventually be able to handle.

Use of different strategies or mental models can be accommodated in an inference network. This can take the form of either a single strategy/mental model choice for all tasks in a class, as studied by Mislevy and Verhelst (1990), or strategy/model switching from one task to another (as in Snow & Lohman, 1984). The nature and the strength of inferences one can draw will depend on the potential observational settings. With rich information, such as verbal protocols or partial solutions, it may be possible to characterize the range of solution methods the student has available and the conditions under which he employs them.

Testing "higher-order thinking" can be accomplished by including unobservable nodes for connections among more basic facts or concepts, and observable nodes that correspond to tasks for which the relationships of interest are critical. Because such tasks might well be open-ended and approachable in a variety of ways, the possibility of alternative solution strategies would need to be built into the network.

Adaptive testing can be carried out among concepts, not just for a single concept. IRT applications of adaptive testing are based on the one-to-many relationships that are appropriate for determining overall levels of proficiency, but inadequate for understanding connections among concepts. The inference network facilitates stepping variously throughout a domain, gathering information about critical domains by presenting tasks that call for varying combinations of key skills.

Handling atypical knowledge configurations or observational patterns can be accomplished by incorporating nodes analogous to the "Other" disease state in MUNIN or the catch-all IRT class in Yamamoto's (1987) *Hybrid* model. An "Other" state of understanding is a mechanism for capturing observational patterns that do not accord with

those specifically built into the network. A situation-sensitive student report might be generated in an instructional system when such a node becomes prominent, signalling that more intelligence than is embodied in the system is needed to figure out what this student is doing, and decide what to do about it.

Conclusion

Learning can be enhanced by a unified conceptual framework for instruction, testing, and reporting, because only in such a framework can coherent feedback loops be constructed. This presentation has focused on the educational measurement aspect of a system built on this premise. The recent introduction of measurement models built around states of understanding, and of inferential techniques to connect such pieces into networks that describe domains of school learning, provide a foundation for improved educational practice in this manner.

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Appendix

Equations for the Latent Class Model

The Model

Let $\eta = (\eta_0,...\eta_4)$ denote the stage of understanding of a child, with $\eta_k=1$ if he or she is in Stage k and 0 if not. Let $\pi = (\pi_0,...,\pi_4)$ denote the population proportions of children in these classes; that is, $\pi_k \equiv p(\eta_k=1)$. Let x_j represent a response to Task j, 1 if correct and 0 if not; j runs from 1 to 24. The conditional probabilities of correct response are $Prob(x_j=1|\eta_k=1)$, or P_{jk} for short. **P** denotes the matrix $((P_{jk}))$. A vector of item responses, $\mathbf{x} = (x_1,...,x_{24})$ is assumed to have the following probability *conditional on Stage membership*:

$$p(\mathbf{x}|\eta_k=1) = \prod_j P_{jk}^{x_j} (1-P_{jk})^{1-x_j} . \qquad (1)$$

Similar expressions are assumed to hold for subsets of responses as well, regardless of the order in which they are observed.

The *marginal* probability of a response vector is an average of terms like (1), weighted by the population probabilities of stage membership:

$$p(\mathbf{x}) = \sum_{k=0}^{4} p(\mathbf{x}|\eta_k=1)\ \pi_k. \qquad (2)$$

Let **X** denote the matrix of response vectors of a sample of N respondents. For a generic pattern $\mathbf{x}_\ell$, let n_ℓ be the number of respondents producing this pattern. The probability of **X** as a function of **P** and π has the form

$$P(\mathbf{X}|\mathbf{P},\pi) = C \prod_\ell p(\mathbf{x}_\ell)^{n_\ell} , \qquad (3)$$

where C does not depend on **P** or π. Once **X** has been observed, (3) can be interpreted as a likelihood function, and maxima may be found with respect to **P** and π.

Because N is only 120 in the balance beam example, a number of constraints were introduced so that stable estimates would be obtained. Many could be relaxed or removed

with larger samples. The results reported in Table 2 represent the best-fitting result among several models with similar numbers of constraints. The P_{jk}s that appear as .333 in Table 1 were fixed at that value. All four items of a given type were constrained to have the same P_{jk}s. For a given column, all P_{jk}s in cells that correspond to 1's in Table 1 were constrained to be equal to a single estimated value. Any cells in that column that correspond to 0's were constrained to its complement.

Adaptive Testing

The maximum likelihood estimates of **P** and π were treated as known true parameter values during simulated adaptive testing. The uncertainty in these values could be taken into account, but we have avoided the complication for this demonstration.

Before observing any responses from a given child, the expected value of his η is the population value π. The expected value of a response to a particular item j is obtained analogously to (2), simplified to a single, as yet unobserved, response:

$$p(x_j=1) = \sum_k p(x_j=1|\eta_k=1)\, p(\eta_k=1)$$

$$= \sum_k P_{jk}\, p(\eta_k=1)\,. \qquad (4)$$

Suppose that Item g is administered to a particular examinee, and the value of x_g, either 0 or 1, becomes known. How is this information propagated through the network? First, using Bayes theorem, we update probabilities for his η. For k=0,...,4,

$$p(\eta_k=1|x_g) = \frac{p(x_g|\eta_k=1)\, p(\eta_k=1)}{\sum_h p(x_g|\eta_h=1)\, p(\eta_h=1)}\,. \qquad (5)$$

This gives new probabilities that the examinee is in each of the possible stages. These are in turn reflected in new expectations for items not yet administered by replacing $p(\eta_k=1)$ in (4) with $p(\eta_k=1|x_g)$ to obtain

$$p(x_j=1|x_g) = \sum_k p(x_j=1|\eta_k=1)\, p(\eta_k=1|x_g)\,. \qquad (6)$$

This process can be repeated with additional items presented one at a time. Let $\mathbf{x}_s$ represent a partial response sequence; Item s+1 is next administered to form $\mathbf{x}_{s+1}$. Then

$$p(\eta_k=1|\mathbf{x}_{s+1}) = \frac{p(x_{s+1}=1|\eta_k=1)p(\eta_k=1|\mathbf{x}_s)}{\sum_h p(x_{s+1}=1|\eta_h=1)p(\eta_h=1|\mathbf{x}_s)} , \tag{7}$$

and, for items not yet presented,

$$p(x_j=1|\mathbf{x}_{s+1}) = \sum_k p(x_j=1|\eta_k=1)\ p(\eta_k=1|\mathbf{x}_{s+1}) . \tag{8}$$

Selecting which item to present next and deciding when to stop depends on probabilities for η. In this paper we have addressed only the case in which no decision-making cost structure is available, and we address only the goal of minimizing uncertainty about η. This can be accomplished by *minimum entropy* adaptive testing. Entropy is a measure of randomness. For the five-class balance beam problem, the maximal value of entropy occurs when probabilities of all five classes are equal, and the minimal value occurs when the probability of one particular stage is one. The general formula for entropy after having observed $\mathbf{x}_s$ is

$$E(\mathbf{x}_s) = -\sum_k p(\eta_k=1|\mathbf{x}_s) \log[p(\eta_k=1|\mathbf{x}_s)] . \tag{9}$$

After having observed $\mathbf{x}_s$, one can evaluate the expected entropy associated with the administration of any remaining item j as

$$E\left[\mathbf{x}_s \cap (x_j=0)\right] p(x_j=0|\mathbf{x}_s) + E\left[\mathbf{x}_s \cap (x_j=1)\right] p(x_j=1|\mathbf{x}_s) \tag{10}$$

The item that minimizes (10) is presented next.

It bears repeating that these formulae assume both that the model is correct and the conditional probabilities are known with certainty. Violations of these assumptions generally degrade knowledge about an examinee's state, making (5) and (8) in particular overly optimistic. Work remains to be done, in studying the robustness of the approach to violations of the assumptions, learning how to minimize violations in practice, and modifying the model or the conditional probabilities to mitigate inferential errors in the presence of violations.

TABLE 1

Theoretical Conditional Probabilities—
Expected Proportions of correct Response

Problem type	Stage 0	Stage I	Stage II	Stage III	Stage IV
E	.333	1.000	1.000	1.000	1.000
D	.333	1.000	1.000	1.000	1.000
S	.333	.000	1.000	1.000	1.000
CD	.333	1.000	1.000	.333	1.000
CS	.333	.000	.000	.333	1.000
CE	.333	.000	.000	.333	1.000

TABLE 2

Estimated Conditional Probabilities—
Expected Proportions of correct Response

Problem type	Stage 0	Stage I	Stage II	Stage III	Stage IV
E	.333*	.973	.883	.981	.943
D	.333*	.973	.883	.981	.943
S	.333*	.026	.883	.981	.943
CD	.333*	.973	.883	.333*	.943
CS	.333*	.026	.116	.333*	.943
CE	.333*	.026	.116	.333*	.943

* denotes fixed value

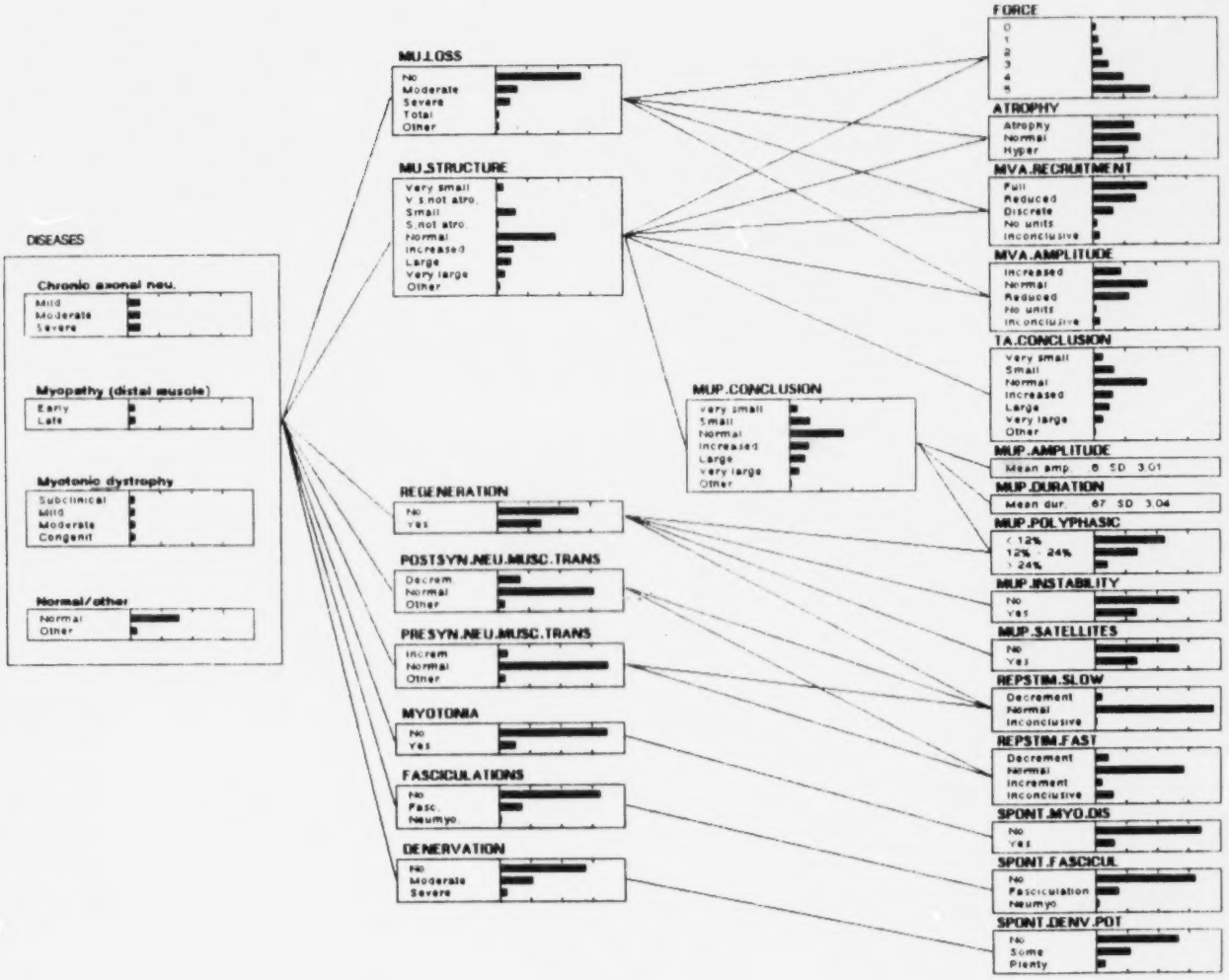

FIGURE 1

The MUNIN Network: Initial Status

(From Andreassen et al., 1987)

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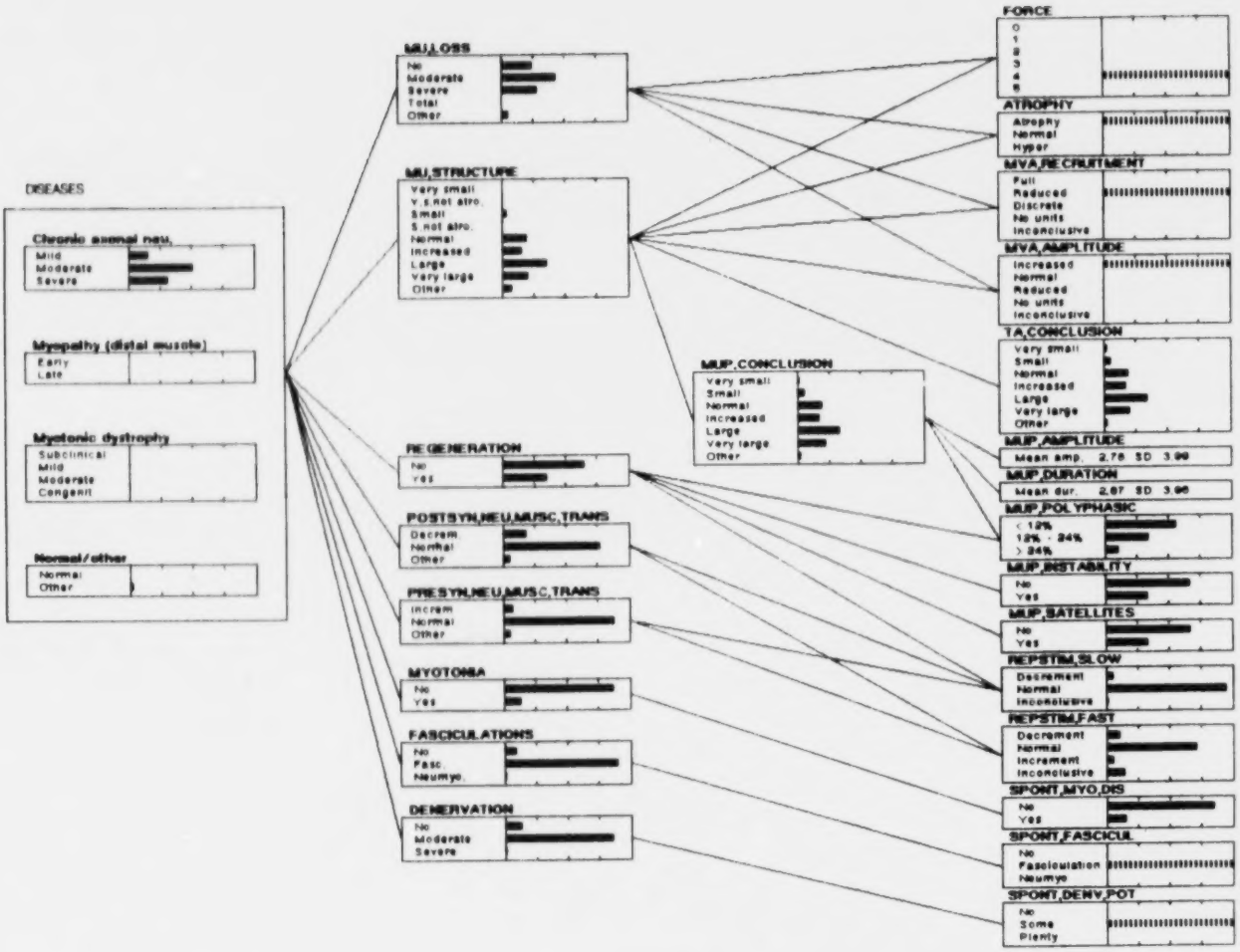

FIGURE 2

The MUNIN Network: After Selected Observations

(From Andreassen et al., 1987)

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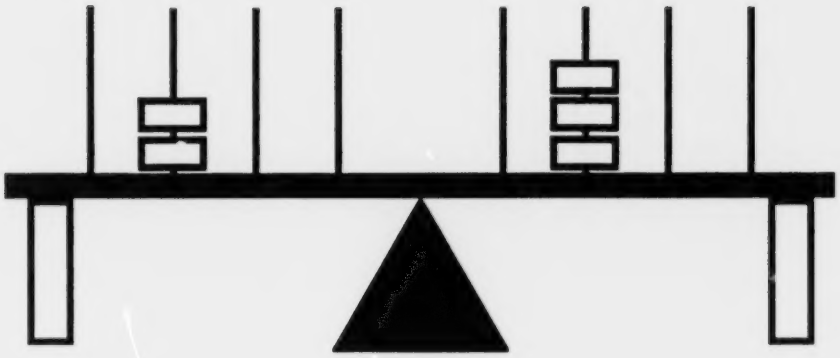


When the blocks are removed, will the beam tip left, tip right, or stay flat?

Figure 3

A Sample Balance-Beam Task

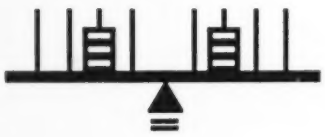
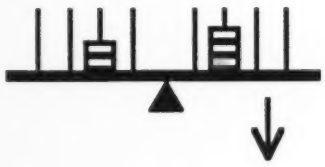
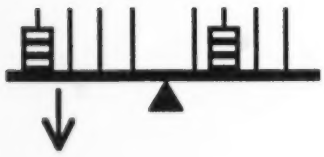
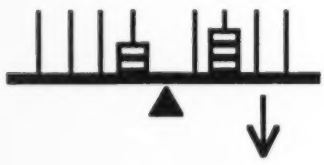
Item Type	Sample Item	Description
E		**Equal problems** (E), with matching weights and lengths on both sides.
D		**Dominant problems** (D), with unequal weights but equal lengths.
S		**Subordinate problems** (S), with unequal lengths but equal weights.
CD		**Conflict-dominant problems** (CD), in which one side has greater weight, the other has greater length, and the side with the heavier weight will go down.
CS		**Conflict-subordinate problems** (CS), in which one side has greater weight, the other has greater length, and the side with the greater length will go down.
CE		**Conflict-equal problems** (CE), in which one side has greater weight, the other has greater length, and the beam will balance.

Figure 4

Sample Balance Beam Items

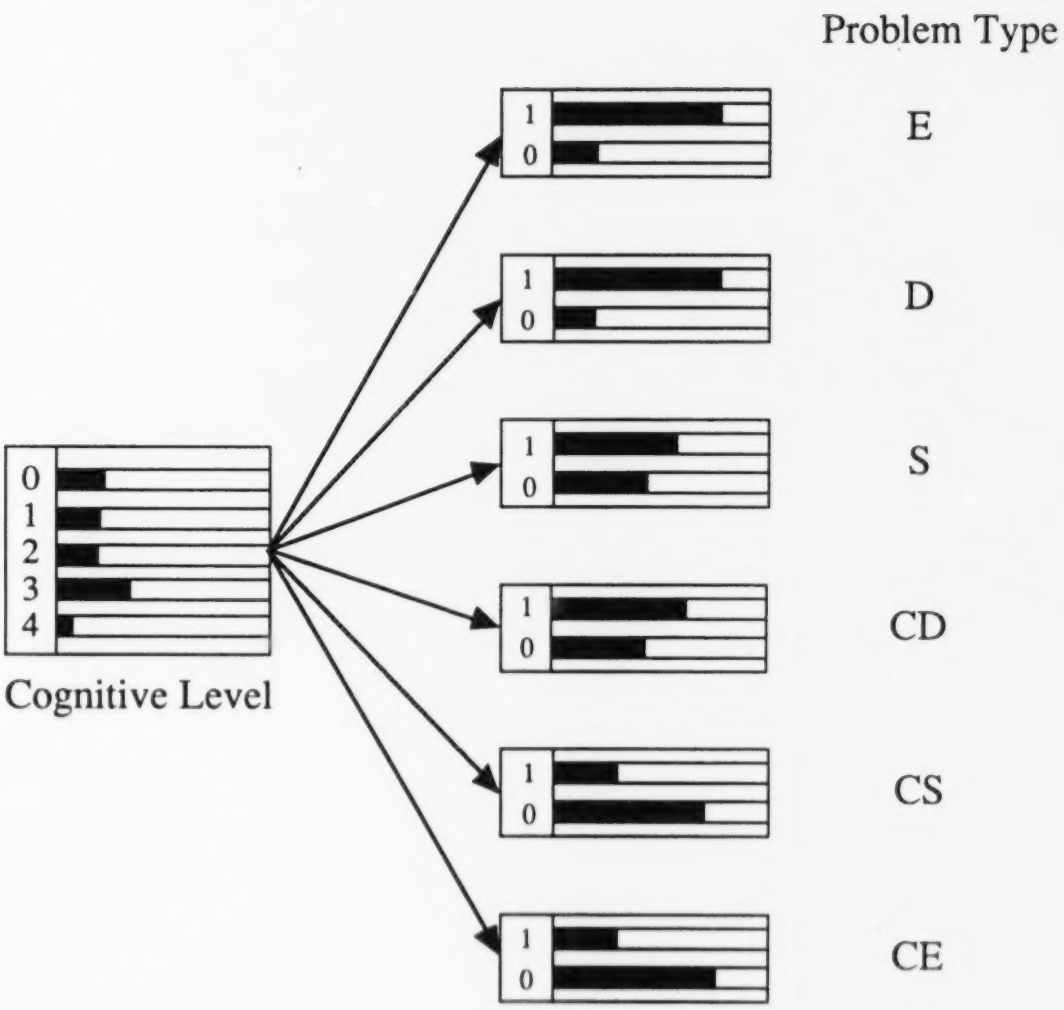


Figure 5
Initial State in an Inference Network
for the Balance Beam Example

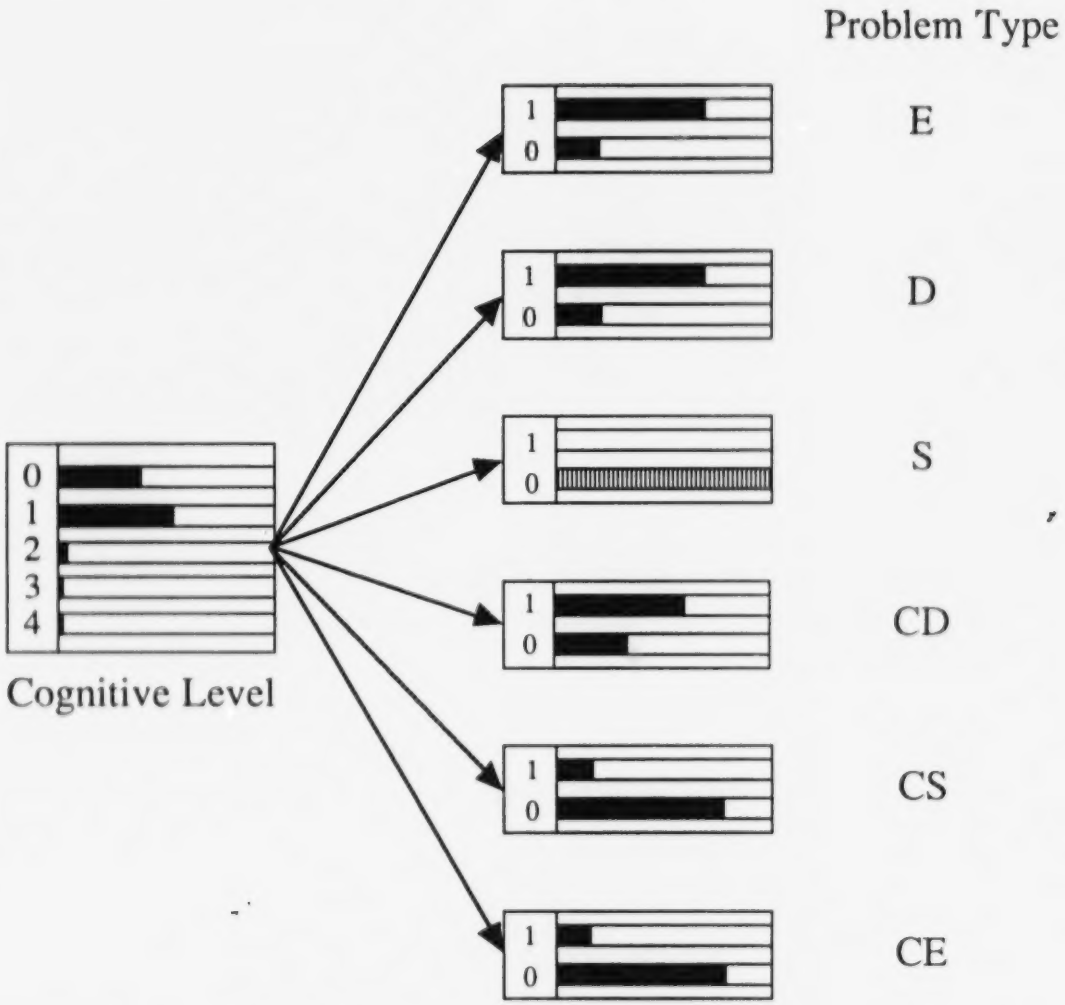


Figure 6

State of Knowledge about Cognitive Level

after an Incorrect Response to an S Item

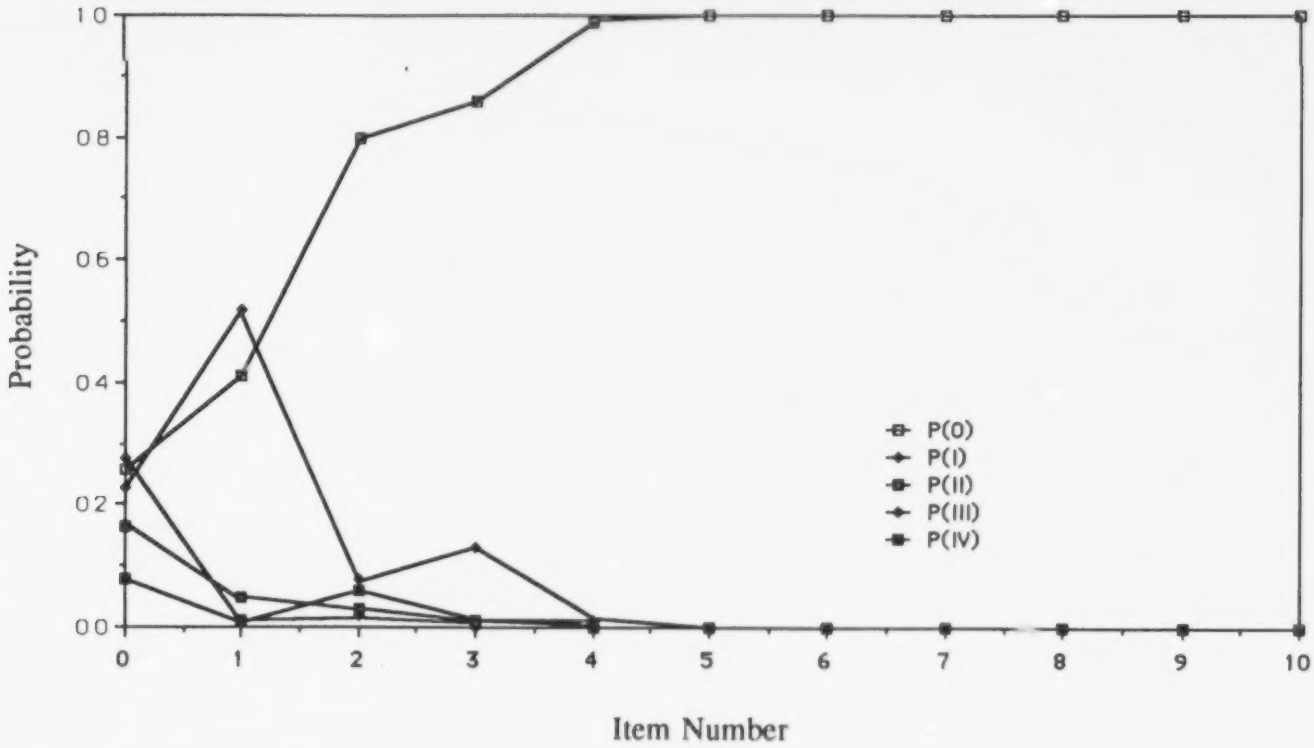


Figure 7

Posterior Probabilities of Cognitive Levels

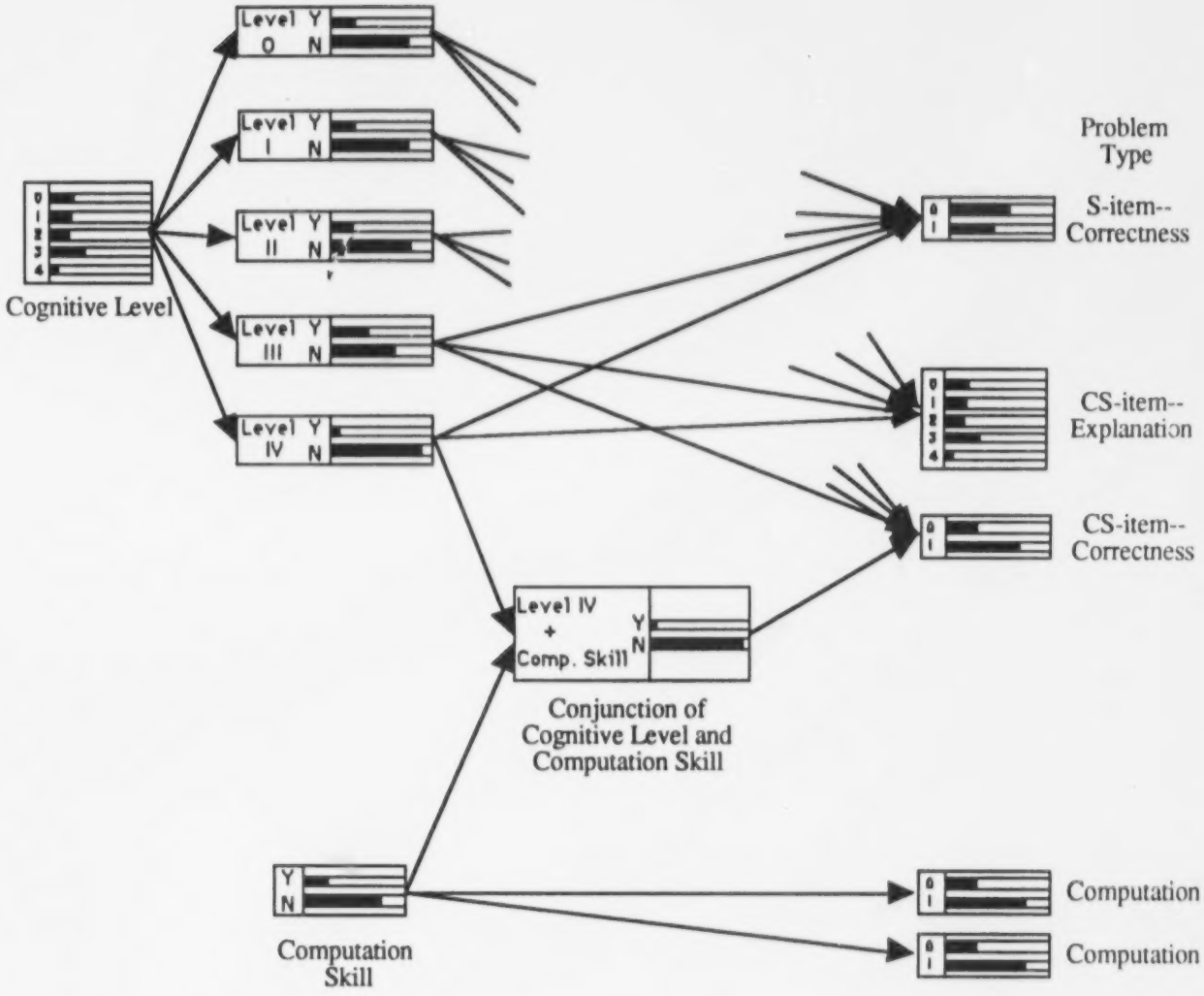


Figure 8

Representation of an Extended Balance-Beam Network

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